REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (*Leave Blank*)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	October 1997	Final report 9/1/94 - 10/31/97

4. TITLE AND SUBTITLE

A New Approach to Radar Waveform Design

5. FUNDING NUMBERS

F49620-93-1-0160
AFOSR-TR-97
0618

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9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

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NR

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

19971204 198

11 SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Distribution Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT (*Maximum 200 words*)

Producing families of codes that behave like finite Gaussian white noise is one of the requirements for some modern CDMA communication systems. This noise-like behavior should be obtained jointly with a good cross-correlation performance of the codes. Finally they must be easy to generate without high computer memory costs. This work was done in close collaboration with Hughes research Laboratory and Hughes Space & Communication Co. Our research was reported in detail to the National Security Agency. We have constructed a new class of codes based on new mathematical tools using properties of mixing ergodic transformation having better low cross-correlation as compared with Gold codes and very good spread spectrum properties. This research has led to a patentbeing filed by Hughes Research Laboratory.

14. SUBJECT TERMS	15. NUMBER OF PAGES
CDMA communication, Digital signal processing	5
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

1 Objective

Our proposal to DARPA was to study the use of the Weil Transform to construct waveforms that are suitable for a multiscale analysis of sonar and radar imagery embedded in clutter and perhaps even speckle. This study of the ambiguity function of signals leads to an analysis of auto- and cross-correlation properties of signals. This suggested a detour into the role that ergodicity and especially mixing ergodic transformations could play in the creation of codes for Code Division Multiple Access (CDMA) systems. With the agreement of Dr. A. Tsao of DARPA, we made such a study a part of our goal.

2 Status of Effort

A mathematical theory of high accuracy multi-wavelets of short supports has been developed leading to construction and classification of such multi-wavelets. Through this effort, taken in collaboration with Hughes Research Laboratory and Professor R. Coiffman et. al. of Yale University, we produced a structured theory to explain and better understand the heuristics of our earlier numerical results.

Since it was important to us to try to understand why these codes worked, our goal has been to develop a new class of codes that have advantages over Gold codes for CDMA and digital radar. Mathematically the idea has been to use some classical results of H. Weyl to modify already studied codes to enhance various measures of goodness of codes. This activity was directed toward the investigation of properties of mixing ergodic transformations and to the development of new mathematical tools to understand their use in the generation of sequences of pseudo-random transmission of codes.

3 Accomplishments and New Findings

The object of developing CDMA covert codes is to produce finite codes to resemble finite Gaussian white noise. There are a series of tests that are used to measure success. Our codes have met the challenge of this test. We presented our codes at a meeting at National Security Agency (NSA).

Although some of the original results based on ergodic transformations were not competitive with ones found in the classical literature, the stronger hypothesis of mixing ergodic transformations led to numerical experiments which demonstrated the significance of this stronger asymptotic theory on the construction of codes having better low cross-correlation as compared with Gold codes and very good spread spectrum properties. A patent has been filed by Hughes Electronics Corporate Patents and Licenses Office under the title :

New CDMA Code Generator Employing Mixing Ergodic Transformations.

Several papers developing the mathematical theory underlying these efforts have been produced.

- Artur Sowa, *High Accuracy Multiwavelets with Short Supports*, submitted to IEEE TRANSACTIONS ON SIGNAL PROCESSING

  This paper shows how to construct multiwavelets using wavelets and certain unitary group actions. A large class of new orthogonal multiwavelets with desirable features is constructed in this way. The above mentioned group actions might find application to the problem of clatter extraction.

- Frank Geshwind, Artur Sowa, *Classifying short multiwavelets*, in preparation

  In this paper written jointly with Frank Geshwind (Yale University), we classify maximal accuracy biwavelets on $[0, 2]$ up to (gauge-) $SL(2, R)$-group action. Work is continued in hope of dropping the requirement for accuracy to be maximal.

- "Transmission Codes and Bernoulli transformations", to appear in "Applied and Computational Harmonic Analysis".

  Our activity was directed toward the investigation of properties of mixing ergodic transformations We started the development of new mathematical tools to understand their use in the generation of sequences of pseudo-random transmission codes. We determined a series of properties that were satisfied by Bernoulli Transformations that justify their use as a new tool for code generation. Our results were exposed in the paper "Transmission Codes and Bernoulli transformations", to appear in "Applied and Computational Harmonic Analysis".

  In collaboration with Dr. U. von der Embse from Hughes Space&Comm we started the study of the problem of optimizing the performance of a new waveform to be utilized in combination with our codes. This new "Wavelet Finite Impulse Response" filter design seems to allow higher efficiency in the reconstruction of the signal than known techniques.

  We were interested in the study of the cyclostationarity properties of a series of LPD waveforms.

4 Technical Publications

- L. Auslander, P.E. Barbano, "Communication Codes and Mixing Ergodic Transformations," to appear in *Journal for Harmonic Analysis and Applications*

- L. Auslander, P.E. Barbano, "An Ergodic Approach to Signal Set Design," submitted for publication to *IEEE Trnas. in SP.*

- L. Auslander and F. Warner, "Radar Waveform Design and the Heisenberg Group," to appear in *Applied and Computational Harmonic Analysis.*
- L. Auslander and F. Warner, "Weil Multipliers," to appear in *Journal of Fourier Analysis and Applications.*
- Artur Sowa, "High Accuracy Multiwavelets with Short Supports," submitted to IEEE TRANSACTIONS ON SIGNAL PROCESSING.
- Frank Geshwind and Artur Sowa, "Classifying short multiwavelets," in preparation.

5 Interactions/Transitions

- Meeting of Board of Mathematical Sciences NAS.
- In Paris in June 1995, L. Auslander held extensive discussions with Y. Meyer and other French mathematicians.
- Attended meetings organized by Dr. Nachman Kriklawn AFB, July 19-20, 1995.
- The results of the work on CDMA code and mixing ergodic transformations were presented at the DARPA Computational Mathematics Program Review, session 1: Signal/Image Processing, held at Strategic Analysis Inc., Arlington, VA on 6/10/1996.
- We have been working closely with Hughes Research Laboratory and Hughes Space & Comm. at HRL. The collaborations has been with
  - Roy M. Matic
  - Xiang-Gen Xia
  - Urban Von der Embse.

  We have presented some of our results to a group of scientists at Hughes led by Doc Dougherty.

6 Patent Disclosures

A patent application was filed by the Hughes Electronics Corporate Patents & Licensing office under the title

New CDMA Code Generator Employing Mixing Ergodic Transformations. The inventors listed in the Patent Disclosure (PD-95474) are:

- Louis Auslander (CUNY-Graduate Center)

- Paolo Emilio Barbano (CUNY-Graduate Center)
- Urban Von der Embse (Hughes Space & Communication)
- Xiang-Gen Xia (Hughes Research Laboratory)
- Roy M. Matic (Hughes Research Laboratory)